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Work n°1: Material Frame-Indifference

Non-linear Mechanics of Materials

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Unit	:	Constitutive theory for large deformations
Learning goal	:	Simulate the kinematics of large deformations continuum bodies rigorously using the constitutive theory of continuum mechanics

Evaluation description

In this activity, the student will be capable of discover the importance of the principle of material frame-indifference (or objectivity) when it is applied to an incremental context. The theory behind the principle is very abstract, so the implementation (integration) of objective kinematics in a numerical framework requires some non-trivial assumptions which will be undercover in this work.

Stage I: Theory

de Montleau et al. (2008) (available in UVirtual) discussed and proposed a method in which the kinematics are integrated into a discrete framework while maintaining the principle of material frame-indifference (objectivity). Sections 1 and 2 of de Montleau et al. (2008) contain the introduction, the literature review and theoretical preliminaries.

1. After reading section 1 and 2, describe the scope (which aspects of the incremental objectivity are important) and motivation (why incremental objectivity is worth to study).
2. Why are there different methods for calculating the velocity gradient in the literature? Explain how these methods are linked with the application of the principle of frame-indifference in a numerical framework. Refer to Cescotto (section 2.11 1992) for another view of the same topic.
3. Explain mathematically how the *incremental deformation gradient*, Equation 7, is obtained. Why it is useful? Hint: check Cescotto (section 2.11.4.2 1992) or Simo and Hughes (1998, section 8.1.1).

Stage II: Implementation

The previous concepts are now applied to the kinematics of a deformable body. For this, consider a (2D) squared element in a initial (reference) configuration defined by the vertices $A = (0,0)$, $B = (1,0)$, $C = (1,1)$ and $D = (0,1)$ at $t = 0\,\mathrm{s}$. A *simple shear* motion is given by $\boldsymbol{x} = \boldsymbol{\chi}(\boldsymbol{X}, t) = \boldsymbol{X} + \tan\theta(t)(\boldsymbol{e}_2 \cdot \boldsymbol{X})\boldsymbol{e}_1$ (Holzapfel, 2000, page 93), where $\theta(t) = \dfrac{\pi t}{18}$. A time step is defined in $[t_n, t_{n+1}]$, where t_n is the time at the beginning of the step, t_{n+1} is the time at the end of the step and $\Delta t := t_{n+1} - t_n$ is the time increment.

1. Define a global and local reference axis and the transformation laws between them.
2. For this motion, calculate the local and global deformation gradient. Analyze if the principle objectivity of the tensor is fulfilled.

3. Using a scientific software (Matlab, Python, etc.), calculate for each time increment $\Delta t = 1\,\mathrm{s}$, $\Delta t = 0.5\,\mathrm{s}$, $\Delta t = 0.25\,\mathrm{s}$, apply the definition of the incremental deformation gradient and calculate the coordinates of C and D of the deformed shape until $t = 3\,\mathrm{s}$:
   - Respect to the **initial** configuration (global axis).
   - Respect to the configuration defined at t_n (local axis).
   - Discuss, are they the same? if not, how they could be the same? Discuss in terms of the global and local frames, frame rotations and the objectivity requirements.
4. The strains increments for each method discussed in de Montleau et al. (2008).

Stage III: Validation

Some commercial finite element software integrate the kinematics in the non-linear regime using Hughes's assumption. Nevertheless, commercial softwares usually work as *black boxes* so the user has no access to the base code in order to validate the used method. In this stage, the student should perform a simple shear deformation (as in the previous section) in a single finite element using the `PLANE182` element in Ansys Mechanical APDL, in order to validate the results obtained in the previous sectiion i.e. compare the results obtained from the implementation and from Ansys Mechanical.

1. Review the theory reference of Ansys Mechanical and find which method(s) is(are) used to calculate the strain increments.
2. Simulate a plane element undergoing the same simple shear deformation as in the point 2. Select a linear elastic material model ($E = 29.0\,\mathrm{GPa}$, $\nu = 0.15$, $\alpha = 11 \times 10^{-6}$) and repeat 2(a). Analyze the results as in 2(a).
3. Repeat the simulation of 3(b) using an iso-thermal Neo-Hookean hyperelastic material model and with the `NLGEOM` option activated.
   - Describe the observed differences and analyze the results, comparing the obtained results with and without `NLGEOM`.
   - Give a theoretical explanation for the observed differences based on the paper of de Montleau et al. (2008) and Cescotto (1992).

References

Cescotto, S. (1992). "Finite Deformation of Solids". In: *Numerical Modelling of Material Deformation Processes: Research, Development and Applications*. Ed. by P. Hartley, I. Pillinger, and C. Sturgess. London: Springer, pp. 20–67.

de Montleau, P., A. M. Habraken, and L. Duchêne (2008). "A New Finite Element Integration Scheme. Application to a Simple Shear Test of Anisotropic Material". In: *International Journal for Numerical Methods in Engineering* 73.10, pp. 1395–1412.

Holzapfel, G. A. (2000). *Nonlinear Solid Mechanics : A Continuum Approach for Engineering*. Wiley.

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