DEPARTAMENTO DE INGENIERÍA EN OBRAS CIVILES
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Work n°2: Viscoelasticity

Non-linear Mechanics of Materials

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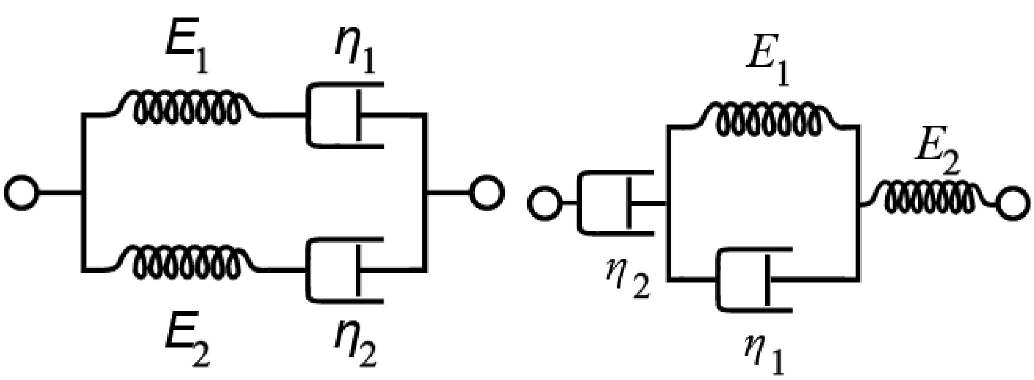
Unit	:	Non-linear viscoelasticity
Learning goal	:	Relate the viscoelastic phenomena of materials to non-linear constitutive models based on internal variables, implementing robust numerical algorithms in scientific softwares.

Evaluation description

In this activity, the student will be capable of formulating (thermodynamically consistent) 3D non-linear viscoelastic models though the generalization of a given 1D analogy model. The kinetic equation can be formulated as an ordinary differential equation (ODE), whose solution governs the evolution of the internal variable(s). The material parameters, as well as other viscoelastic parameters, should be interpreted for the numerical modeling using the finite element method.

Stage I: Theory

The Burgers model is a four-parameter model which exhibit important features of viscoelasticity, such as elastic deformation, creep and recovery (Chemical Retrieval on the Web (CROW), 2020). The arrangement is depicted in Figure 1.



(a) Maxwell representation.

(b) Kelvin-Voigt representation.

Figure 1: Burgers model (Wikipedia contributors, 2022).

Considering this (analogy) model and the work of Majda and Skrodzewicz (2009),

1. Discuss why this model is selected, in terms of its strengths and shortcomings.

2. Given a certain load, obtain numerical solutions for stress relaxation $\sigma(t)$ and creep $\varepsilon(t)$. Note: consider the different variations of the Runge-Kutta method avaiable in scientific softwares.

3. Formulate a thermodynamically consistent 3D viscoelastic model based on Burgers' analogy model.

Stage II: Implementation

Consider a (2D) squared element in a initial (reference) configuration defined by the vertices $A = (0, 0)$, $B = (1, 0)$, $C = (1, 1)$ and $D = (0, 1)$ at $t = 0\,\mathrm{s}$. A time step is defined in $[t_n, t_{n+1}]$, where t_n is the time at the beginning of the step, t_{n+1} is the time at the end of the step and $\Delta t = t_{n+1} - t_n$ is the time increment. Then, we define these two motions:

- A *uniaxial tensile* motion, given by $\boldsymbol{x} = \boldsymbol{\chi}(\boldsymbol{X}, t) = \boldsymbol{X} + \dfrac{t}{3}(\boldsymbol{e}_1 \cdot \boldsymbol{X})\boldsymbol{e}_1$, until $t = 3\,\mathrm{s}$.
- A *simple shear* motion (Holzapfel, 2000, page 93), given by $\boldsymbol{x} = \boldsymbol{\chi}(\boldsymbol{X}, t) = \boldsymbol{X} + \tan\theta(t)(\boldsymbol{e}_2 \cdot \boldsymbol{X})\boldsymbol{e}_1$, where $\theta(t) = \dfrac{\pi t}{18}$, until $t = 3\,\mathrm{s}$.

Using the material parameters shown in the work of Majda and Skrodzewicz (2009), and for these two motions:

1. Obtain the stress-strain curves.
2. Compare the small (infinitesimal) strain versus large strain version of the chosen viscoelastic model.
3. For the large strain version, discuss how the frame-indifference principle is guaranteed during the motion.
4. Evaluate the influence of the time step calculated at $\Delta t = 1\,\mathrm{s}$, $\Delta t = 0.5\,\mathrm{s}$, $\Delta t = 0.25\,\mathrm{s}$ in the stress-strain curve.

Stage III: Validation

Using Ansys Mechanical APDL, perform a 2D homogeneous deformation in a `PLANE182` element of the motions presented in Stage II. Then:

1. Ansys uses Prony series to include material parameters in viscoelastic models. Explain this approach and how it is related with the work of Majda and Skrodzewicz (2009).
2. Compare the small (infinitesimal) strain versus large strain version of the chosen viscoelastic model.
3. Evaluate the influence of the time step calculated at $\Delta t = 1\,\mathrm{s}$, $\Delta t = 0.5\,\mathrm{s}$, $\Delta t = 0.25\,\mathrm{s}$ in the stress-strain curve.
4. Compare with the results obtained in Stage II and analyze the observations.

References

Chemical Retrieval on the Web (CROW) (Aug. 27, 2020). *Viscoelastic Models for Linear Viscoelastic Response*. polymerdatabase.com. URL: https://polymerdatabase.com/polymer%20physics/Linear%20Viscoelasticity.html (visited on 10/12/2022).

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Majda, P. and J. Skrodzewicz (June 1, 2009). "A Modified Creep Model of Epoxy Adhesive at Ambient Temperature". In: *International Journal of Adhesion and Adhesives* 29.4, pp. 396–404.

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